Modeling the Effect of Income Segregation on Communicable Disease Transmission

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Abstract

Income segregation has been on the rise in developing countries, where many communicable diseases are still prevalent. This study investigates how income segregation affects communicable disease transmission through the development of a novel model that includes income segregation and individuals' health-seeking behavior. The general model proposed here assumes that health of an individual is affected by 1) the health-seeking behavior of individuals (e.g., going to the hospital, taking medicine); 2) the communal health stock; and 3) exposure to the communal health stock. The communal health stock is comprised of the amenities that make people healthier in a given community (e.g., the number of health-care facilities). Income segregation is defined here as a combination of income inequality and residential segregation, which exists when some people have higher exposure to the communal health stock than other people. In this model, income segregation exists when poor people disproportionately live in neighborhoods with lower exposure to the communal health stock than rich people. A decrease in income segregation means the income of the poor increases along with their exposure to the communal health stock. The general model applied here predicts that an increase in the poor's income will increase their health by enabling them to afford more health-seeking behavior and finds that higher exposure to the communal health stock directly increases the health of the poor. Higher exposure to the communal health stock, however, is found to decrease the poor's health-seeking behavior, which reduces their health. The general model finds, therefore, that a decrease in income segregation will have an ambiguous effect on the health of individuals and the overall community. Probing further, the analysis replaces the general model with a more specific model, which predicts that overall, a decrease in income segregation increases individual health. Furthermore, in the more specific model, it is possible that the poor get stuck in a low-health equilibrium while the rich stay in a high-health equilibrium.

Introduction

Income segregation, in which gated communities separate the rich and the poor, has been rising in developing countries, where communicable diseases are still prevalent.[1][2] Avoiding disease outbreak is one of the most important goals of public health. This paper develops a novel

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[1] For instance, a high rise tower called Hamilton Court in Gurgagon, India protects its residents from surrounding slums with fences, security guards, a water system, a private school, and a private hospital (Sengupta, 2008). In China, rural residents migrate to cities to seek employment, while urban citizens move to suburban gated communities and shield themselves from the "uncivilized" population (Pow, 2007; Wu, 2005). In Brazil, the population living in close condominiums (CCs) doubled between 1998 and 2002 (Silva, 2007). These enclaves locate mainly around city suburbs and offer millions of middle- to upper-class Brazilians high-quality homes, secured neighborhoods, a clean environment, and a sophisticated leisure infrastructure. It is evident that income segregation is a global trend.

[2] Income segregation may also affect noncommunicable diseases (e.g., diabetes, lung cancer, etc) and other broader health outcomes. In this paper, I examine communicable diseases.

theoretical model that examines how income segregation affects communicable disease transmission. Understanding the link between the two can inform the thinking of public health officials in the development of policy and allocation of resources needed to mitigate the transmission of communicable diseases and thereby protect public health.

The model proposed here mostly applies to dense urban areas in developing countries, where public health systems are weak and spatial health externalities are prevalent because of communicable diseases. In the proposed model, an individual maximizes his or her utility (happiness) by choosing between health and other consumptions. A person's health is affected by three inputs: 1) health-seeking behavior, 2) communal health stock,[3] and 3) exposure to the communal health stock. Health-seeking behaviors are any behaviors that individuals take to improve their health. Some examples of health-seeking behaviors are going to the hospital, taking medicine, and getting vaccinated. Communal health stock is comprised of things in the community that are good for an individual's health, for examples, the number of hospitals or health-care facilities, the availability of clean water, and the number of healthy residents in the community. The communal health stock is modeled as a weighted average of individuals' health. It is assumed that higher exposure to the communal health stock means better access to things in the community that are good for an individual's health (such as good health-care services and clean water) and less exposure to things in the community that are bad for an individual's health (such as pollutants). Thus, it is assumed that higher exposure to communal health makes individuals healthier. It is also assumed that a person's health-seeking behavior is more effective in increasing their health when the communal health stock is low (e.g., fewer healthy residents in the community or fewer health-care facilities). Furthermore, it is assumed that if a person is more exposed to the communal health stock, the effect of health-seeking behavior on this person's health is weaker. Though these assumptions may not always be true, they are presumed for the purposes of this analysis. Examples of situations when these assumptions hold are offered in the "Assumptions" section, while the examples of situations when these assumptions may not hold are in the "Model Limitations" section, with some discussion of how this affects the model's results.

In a perfectly equal and unsegregated world, everyone would have the same exposure to the communal health stock and the same income. Income inequality exists, quite simply, when the rich have a higher income than the poor. Residential segregation exists when some people in a community have higher exposure to the communal health stock and other people have lower exposure.[4] Income segregation exists when both income inequality and residential segregation

[3] Note that even though the paper discusses infectious disease transmission, the communal health stock is modeled as good and healthy.

[4] Note that it is possible that the rich and the poor live in different neighborhoods that have the same exposure to the communal health stocks (e.g, the same access to health-care services). The separation between the rich and the poor alone (even without the situation where they have different access to healthcare) can affect an individual's health. However, this paper does not consider how other dimensions of residential segregation could affect health. Rather, it focuses on how exposure to the communal health stock (e.g., accessing health-care services) affects health. Thus, exposure to communal health stock is embedded in the existence of residential segregation.

exist in a way that results in poor people living in a community with less exposure to the communal health stock (e.g., worse access to healthcare) and rich people living in a community with high exposure to the communal health stock (e.g., better access to healthcare). Thus, by this definition, a decrease in income segregation raises the income and exposure of the poor while reducing them for the rich.

The general model proposed here finds that income segregation has an ambiguous effect on an individual's health and thus the communal health stock, depending on whether the effect of the income and exposure to the communal health stock are stronger or an individual's health-seeking behavior effect is stronger. A decrease in income segregation means that the poor have more money and higher exposure to the communal health stock (e.g., better access to healthcare), while the opposite is true for the rich. Specifically, when the poor's income increases, they have more money for health-seeking behaviors, such as taking more medicine, resulting in an increase in their health. When the rich's income decreases, they have less money for health-seeking behavior, resulting in a decrease in their health. Exposure to the communal health stock affects an individual's health directly and indirectly via health-seeking behavior.

Specifically, the direct effect implies that when the exposure to the communal health stock of the poor increases, their health also increases because they now have better access to healthcare. Similarly, when the exposure to the communal health stock of the rich decreases, the health of the rich decreases. The indirect effect of an increase in exposure to the communal health stock via health-seeking behavior suggests that a more exposed individual decreases his health-seeking behavior because it is not rewarded as much or as beneficial as it is for a less exposed person. The reduction in health-seeking behavior as exposure increases decreases an individual's health. Combined, the overall effect of a decrease in income segregation is ambiguous. If an increase in the health of the poor due to an increase in income (decrease in inequality) and an increase in exposure to the communal health (decrease in residential segregation) were to outweigh the decrease in the poor's health due to a reduction in health-seeking behavior, the overall health of the poor would increase. If an increase in the health of the poor due to an increase in income (decrease in inequality) and an increase in exposure to the communal health (decrease in residential segregation) were less than the decrease in the poor's health due to a reduction in health-seeking behavior, then the overall health of the poor would decrease as a result of a reduction in income segregation.

Due to the limitations of the general model to predict the final outcomes, a more specific model that satisfies the general model's assumptions is introduced here. This more specific model predicts that an increase in the health of the poor due to an increase in income (decrease in inequality) and an increase in exposure to the communal health (decrease in residential segregation) is large enough to outweigh a decrease in the poor's health due to a reduction in health-seeking behavior. Thus, the overall health of the poor increases when income segregation decreases. Finally, the model shows that there is a possibility that if the poor do not have enough resources to improve their health in a significant way, they would get stuck in the low-health equilibrium . This result is similar to the poverty trap idea, where some communities or individuals get stuck in poverty if their income is not high enough.

Literature Review

This paper builds on economic models by Wildman (2003) and Momota, Tabata, and Futagami (2005). Momota, Tabata, and Futagami (2005) incorporate health-seeking behavior into an overlapping generation model to explain why some infectious diseases like HIV and syphilis are cyclical. Intuitively, when a disease's prevalence is low, people are more likely to engage in risky behavior, which increases disease prevalence. Wildman (2003) models how the distribution of income affects individuals' health and health inequality, with individual health a function of income and income distribution. With the addition of income distribution, the health production possibility frontier can be nonconcave. Thus, a policy that does not account for income inequality would create unequal health distributions. This paper introduces income segregation in a model with individuals' health and health-seeking behavior (similar to Momota, Tabata, and Futagami, 2005) and income inequality (similarly to Wildman, 2003). It explains how these three components together predict the ambiguous effect of income segregation on health and may generate multiple health equilibria.

This paper relates to the empirical literature on health-seeking behavior in developing countries. Dupas (2011) reviews factors that affect health-seeking behavior in developing countries and finds that information, educational level, financial constraint, and time preferences all play roles in an individual's health-seeking behavior. For example, Madajewicz et al. (2007) and Jalan and Somanathan (2008) find that households in Bangladesh and India, respectively, switch to safer water sources and purification when they learn that their current water sources are contaminated. Additionally, Dupas (2011) shows that adolescent girls change their sexual behaviors when they learn about the risk of getting HIV by type of partner. Tarozzi et al. (2011) and Devoto et al. (2011), in experiments in Orissa, India, and Morocco, find that giving households credits is more likely to increase the likelihood that households adopt health-preventive products such as ITN bed nets and home connections to the drinking-water networks.

This paper also relates to the broad public health and epidemiology empirical literature on the relationship between a community's characteristics—neighborhood income inequality, socioeconomic status, and residential segregation (predominantly racial segregation)—and quality of life. Lynch et al. (2004) provide a review of ninety empirical studies on the association between neighborhood income inequality and health. The units of study are cross-country level, where a country is the unit of observation; within-country cross-sectional level, where a community within a country (city, state, or census tract) is the unit of observation; and multilevel studies, where an individual is the unit of regression and characteristics of the individual's community are taken into consideration. These studies show mixed results: some find negative associations between income inequality and health, some find no associations, and others find that negative associations exist only among subgroups in the population.

Yen and Syme (1999) review the empirical literature on the association between socioeconomic status and health. A majority of studies that do not account for individuals' socioeconomic status report a relationship between a community's socioeconomic status and mortality. Studies controlling for individuals' socioeconomic status find mixed results. In some studies, the association exists, while individuals' variables are stronger predictors of health than environmental variables in others. The rest find that a community's characteristics no longer predict health after accounting for individuals' socioeconomic status.

Regarding empirical results on residential segregation and health, most studies in the United States examine racial segregation. Guest et al. (1998), Polednak (1996, 1991), and LaVeist (1993) find that places in the United States with higher black-white racial segregation have higher mortality rates. Using zip codes in New York City, Fang et al. (1998) find that white people, regardless of age and gender, had higher mortality rates if they lived in black areas rather than white areas. Controlling for socioeconomic status, old blacks living in black neighborhoods actually had lower mortality rates than their counterparts living in white neighborhoods. The mortality rates of young and middle-aged blacks were the same in white and black areas. Regression analyses in Nuru-Jeter and LaVeist (2011) include both racial segregation and income inequality and find that places with higher income inequality experienced lower mortality among whites and higher mortality among blacks. Additionally, income inequality has a stronger negative relationship with mortality in places with higher black-white racial segregation. In terms of income segregation in the United States (not racial segregation), Waitzman and Smith (1998) find that income segregation is associated with higher mortality risks among the poor and lower mortality risks among the elderly rich. Ross et al. (2001) do not find such associations within metropolitan areas (MSAs) in Canada, while the associations do exist within MSAs in the United States.

The theoretical literature explains different pathways through which income inequality may affect health. Wagstaff and van Doorslaer (2000) predict five possible mechanisms: the absolute income hypothesis (AIH), relative income hypothesis (RIH), deprivation hypothesis (DH), relative position hypothesis (RPH), and income inequality hypothesis (IIH). The AIH predicts no association between income inequality and health after accounting for individuals' absolute income. The RIH predicts that income relative to social group average is important. According to the DH, income relative to poverty is important. The RPH argues that an individual's position in the income distribution matters. The IIH suggests that income inequality affects health, even after controlling for absolute income. Mellor and Milyo (2002) introduce strong and weak versions of the IIH. The strong version predicts that an income transfer from the rich to the poor will improve the health of both groups. The weak version suggests that a transfer from the rich to the poor makes the gain of the poor more than the loss to the rich. Thus, society's overall health increases.

In addition, Lynch et al. (2000) offer three interpretations of evidence linking income inequality and health: the individual income, the psychosocial, and the neo-material interpretations. The individual income hypothesis is the same as the AIH described above. The psychosocial hypothesis is similar to the corrosion of social cohesion and promotion of distrust between groups in Ross et al. (2001), which could in turn deteriorate health (Kawachi et al., 1997; Veenstra, 2002). In addition, lack of social cohesion and community attachment may result in underinvestment in health education, medical care (Kaplan et al., 1996), or human resources (Smith, 1996). The neo-material hypothesis in Lynch et al. (2000) states that the direct health effects of income inequality result from the difference of accumulation of exposures that do not result directly from perceptions of disadvantage.

Acevedo-Garcia (2000) explains how residential segregation can affect communicable disease transmission directly and indirectly. Infectious diseases can spread directly in three different ways: spatial distribution, contact patterns between a segregated group and the rest of the population, and density of the susceptible. Since most infectious disease infections require close contact between infectious and susceptible individuals, the spatial distribution of individuals and their movements across spatial locations determine disease spread and persistence. The density of susceptible individuals also affects transmission. Higher densities accelerate transmission because they increase the rates of contact between infectious and susceptible individuals.

Indirect pathways refer to the effect of segregation on the quality of the living environment inhabited by the segregated group (i.e., segregation may result in poverty concentration, overcrowding, housing dilapidation, social disorganization, and limited access to healthcare) (Acevedo-Garcia, 2000; Yen and Syme, 1998). For instance, many studies have confirmed a positive association between low socioeconomic status and tuberculosis (TB) in contemporary populations (Cantwell et al., 1994; Spence et al., 1993). Housing conditions are also important. TB is spread through coughing or spitting by individuals who suffer from active TB. Therefore, physical contact between infected and noninfected individuals and the extent of overcrowding and lack of ventilation are key factors in TB transmission. Another indirect pathway is social disorganization (Acevedo-Garcia, 2000). This leads to risky behaviors such as smoking and drug usage. For instance, crowded housing and coughing associated with smoking increase the chance of contracting TB. In addition, there may be economic consequences of income segregation. Specifically, if poor people are concentrated in one place, that place may collect fewer taxes and, as a result, have poor public transport. Businesses would be less likely to locate in those places, leading to fewer job opportunities. Thus, poor people in poor areas are poorer and cannot afford healthcare (Cooper et al., 2001; Ross et al., 2001).

On the contrary, residential and racial segregation may indirectly benefit health because of strong community and social cohesion or transmission of relevant healthy knowledge. This is because people sharing a similar socioeconomic status, including education and cultural norms, cluster. As a result, social cohesion can provide a supportive effect for the segregated group (Fang et al., 1998; Nuru-Jeter and LaVeist, 2011; Rabkin and Struening, 1976; Yen and Syme, 1999). For example, predominantly black areas have higher levels of black political empowerment and better community and social services (LaVeist, 1992, 1993).

Even though this paper does not study racial segregation, the literature on residential segregation in the United States is often associated with racial segregation. On the one hand, racial segregation and income segregation share many similarities, such as the concentration of poverty and poor socioeconomic status. On the other hand, they do not necessarily coincide. Accounting for income segregation, racial segregation implies that places with predominantly black populations have the same average income as places with predominantly white populations. The protective effects of social cohesion theory in racial segregation suggest that there are both empowerment and psychological benefits from living around people with the same mindset and culture, regardless of economic condition. In other words, blacks living in black neighborhoods would be healthier than blacks living in white neighborhoods when accounting for socioeconomic status. Strictly speaking, black empowerment does not exist in an environment with only income segregation and no racial segregation. Broadly speaking, something like poor empowerment could exist, which may or may not manifest itself in a way similar to black empowerment.

Current Model and Expected Contribution

This model fits in with the public health and epidemiology literature discussed above by examining how income segregation directly affects health, as discussed in Acevedo-Garcia (2000). The paper abstracts the model from the indirect pathways through neighborhood characteristics or socioeconomic status for simplicity. Additionally, the model predicts how income inequality affects health through the absolute income hypothesis explained by Wagstaff and van Doorslaer (2000).

The model is novel because it incorporates the interaction between individuals' health-seeking behaviors and external factors (the communal health stock, exposure to diseases, and income segregation) in a mathematical model to explain how income segregation may affect communicable disease transmission. In addition to external factors such as housing conditions, disease prevalence, and access to healthcare, a person's own health-seeking behavior is also very important to his or her health. For example, health-seeking behavior matters a lot when considering sexually transmitted diseases. All else being equal, the decision of whether or not to get vaccinated also affects whether a person gets the flu. To be clear, the idea that individual health-seeking behavior may affect health exists in the economics discipline (Momota, Tabata, and Futagami, 2005) and in early public health and epidemiology literature (Yen and Syme, 1999). In these frameworks, individual health-seeking behavior is often isolated or is not used to explain how income segregation may affect health. In this model, I allow a feedback loop between external factors, individual health-seeking behavior, and the communal health stock. External factors such as the communal health stock or income segregation affect individual health-seeking behavior, which in turn affects individuals' health and then the communal health stock. This feedback loop creates two predictions. First, the effect of income segregation on health is ambiguous. This ambiguity could partly explain the mixed empirical evidence on the association between income segregation and health provided by Ross et al. (2001) and Waitzman and Smith (1998). Second, it is possible that one segregated community may get stuck in one equilibrium while another is in a different equilibrium. This idea is similar to the poverty trap idea.

Method and Assumption: A General Model

Model Setup

This paper uses a stylized, mathematical model to examine the relationship between income segregation and health. The model makes a number of simplifying assumptions (discussed in detail later), but it allows me to derive clear predictions that can be tested against data in future work.

The model assumes that individuals maximize utility. Utility can be thought of as happiness or satisfaction; the higher a person's utility, the happier he or she is. An individual's utility (U) depends on his or her health level (h) and other consumption goods (c). This relationship is expressed by the mathematical function $U(c,h)$. Utility maximization is a simplifying assumption that makes the model tractable. There is no real-world analog of utility that can be measured or expressed in concrete units, and individuals do not explicitly solve maximization problems when making decisions. However, it is a common assumption in economic models because in many settings, it delivers reasonable predictions about how people behave when they have to make tradeoffs between one activity and another.

It is assumed that a person's health is affected by three factors: his or her health-seeking behavior (x), a communal health stock (H), and exposure to the communal health stock (k). Examples of health-seeking behavior are going to the hospital, taking medicine, and getting vaccinated. The model makes the simplifying assumption that health-seeking behavior can be measured by a single index, x, where a higher value of x indicates more health-seeking behavior. Examples of the communal health stock are fewer sick people and clean water in the community. Again, the model makes the simplifying assumption that the community health stock can be summarized by a single index, H, and the model further assumes that H can be represented as a weighted average of the health of individuals in the community.

Living in a community with a better health stock improves an individual's health. For example, an individual living in a community with good public health facilities (high communal health stock) is more likely to have better health than an identical individual living in a community with poor public health facilities (low communal health stock). In addition, different people in a segregated community might have different access or exposure to the communal health stock. For example, rich people might live closer to good public health facilities, and poor people live further away from good public health facilities. The model conceptualizes this relationship with the variable k, which measures the degree of exposure an individual has to the community health stock. I assume that k takes can range from 0 to 1. A value of k = 0 indicates that individuals derive no benefit from the communal health stock. A value of k = 1 indicates a perfect relationship between the community health stock and individual health. An intermediate value, say k = 0.4, means that 1 unit of the communal health stock H translates into 0.4 units of access to health resources for individuals in the community. Exposure k can vary by income level. These assumptions allow an individual's health to be expressed by the mathematical function h(x,H,k). This function translates levels of individual health-seeking behavior, the community health stock, and individual exposure to the community health stock into a numerical index for an individual's health.

When maximizing utility, a person faces a budget constraint. Specifically, a person only earns a certain amount of income and needs to allocate his money between health-seeking behavior (x) purchasing other consumption goods (c). The model assumes that health-seeking behavior—visiting the doctor, getting vaccines, etc.—has a price of p per unit; thus, total expenditures on health-seeking behavior is px. Consumption of other goods is measured in monetary terms. The model makes the simplifying assumption that there are only two levels of income, I_g and I_r, corresponding to poor and rich, respectively. In the model, income segregation exists when poor people have low exposure to the communal health stock (low k) and rich people have high exposure to it (high k).

Intuitively, to get theoretical predictions, the model assumes that individuals maximize utility by choosing their level of health-seeking behavior (x) and consumption of other goods (c). These choices, combined with the community health stock (H) and degree of exposure to the community health stock (k), determine individual health (h). In turn, the community health stock (H) is determined by averaging the individual health levels (h). Thus, there is a two-way relationship between h and H. Individual health affects the communal health stock, and the communal health stock affects individual health. Mathematically, this gives rise to a system of two equations (individual health as a function of community health, and community health as a function of individual health) that can be solved simultaneously to yield the equilibrium level of both individual and community health.

The remainder of this section sketches the formal setup and solution to the model. Details are provided in an appendix. Formally, the utility maximization problem can be expressed as follows:
Utility function: $U_i = U(c_i, h_i)$ where
U_i: the utility function of person i
c_i: consumption goods of person i, c_i, is between [0;1)
h_i: health production function of person i, h_i, is between [0;1)
k_i: the degree to which person i is exposed to the environment, k_i, is in [0; 1]
x_i: health-seeking behavior, $x_{i,}$ is in [0;1)
H: the communal health stock, defined by $H = \frac{\sum_{i=1}^{n} h_i}{n}$
Budget constraint: $c_i + px_i = I_i$ where
p: the normalized price of health-seeking behavior with respect to consumption goods c_i
I_i: income of person i.

An individual maximizes $U_i[c, h(k_i,\ x_i, H)]$ with respect to c_i and x_i subject to the constraint $c_i +\ px_i\ =\ I_i$. While the community health stock affects individual health, and the community health stock is equal to the average of all individuals' health, it is assumed that communities are large enough that each individual has only a negligible effect on the community health stock. Thus, individuals disregard their impact on H when they maximize utility.

Solving this problem yields the individual's utility maximizing values for health-seeking behavior and consumption of other goods. The paper refers to these optimal values as x_i^* and c_i^*. These utility-maximizing values depend on the individual's income, the price of health-seeking behavior, the communal health stock, and exposure to the communal health stock. From these individual choices of x and c, combined with the community health stock, H, the individual's level of health can be derived. Because x_i^* and c_i^* depend on $I_i, p, and\ H$, individual health h_i^* can be written as a function of these variables as well. That is, the utility-maximizing level of individual health is $h_i^*(I_i, p, H)$. The average value of individual health determines the community health stock, H; thus, $H = \sum h_i^*(I_i, k_i, p, H)/N$, where N is the number of individuals in the community. This equation can be solved for the equilibrium value of H, which I denote by H*.

As mentioned above, there are two levels of income, I_g and I_r, corresponding to poor and rich, respectively. The level of exposure to the community health stock for the poor and rich are denoted by k_g and k_r, respectively. It is assumed that poor people have lower exposure (low k) to the communal health stock and rich people have higher exposure (high k).[5] Note that the communal health stock H is common to both the rich and the poor. The two groups differ not by different levels of H but by different levels of income (I) and exposure (k) to H.

In a perfectly equal world, $I_g = I_r = I$ and $k_g = k_r = k$. When there is income inequality but no residential segregation, $I_g < I_r$ and $k_g = k_r$. When there is residential segregation but no income inequality, $k_g < k_r$ and $I_g = I_r$. When income segregation exists, $I_g < I_r$ and $k_g < k_r$. Thus, income segregation exists when both income inequality and residential segregation exist in a way that results in poor people living in a community with less exposure to the communal health stock (e.g., worse access to healthcare) and rich people living in a community with high exposure to the communal health stock (e.g., better access to healthcare). By this definition, a decrease in income segregation raises the income and exposure of the poor while reducing those of the rich.

Assumptions

This section describes intuitively all the assumptions of the model. The mathematical and technical assumptions can be found in the Appendix.

Assumption 1: The more consumption c one person has and the healthier she/he is, the happier she/he is. In addition, if two people are given one more unit of consumption, *c*, the person with an initial low level of consumption will gain more happiness than the person with an initial high level of consumption. In other words, a person's utility increases at the decreasing rate with respect to consumption c. The model also assumes that consumption and health levels are complementary. If a person is healthy, then a gain in consumption brings him or her more happiness than it does to people who are less healthy.

Assumption 2: An individual's health production function is similar to that of the utility function. Health consists of health-seeking behavior, *x*, exposure coefficient to the environment, *k*, and the communal health stock, *H*. An individual's health level, h, increases, but at decreasing rates with respect to its health-seeking behavior, the exposure coefficient, and the communal health stock.

Assumption 3: When people have almost no health-seeking behavior, a small increase in health-seeking behavior brings a large increase in an individual's health, making them very happy.

Assumption 4: Health-seeking behavior is a smooth function of exposure, income, price, or the communal health stock. In addition, changes in health-seeking behavior when exposure, income, price, or the communal health stock changes are also smooth with respect to these variables.

Assumption 5: The marginal gain in health created by health-seeking behavior *x* in a healthy environment is lower than that in a less healthy environment. This implies that a person living in

[5] The limitations of having only poor and rich people are discussed in "Model's Implication in a More General Situation" section.

a high-health-level community does not need to have much health-seeking behavior and can still be healthy. This means that an individual's health-seeking behavior and the communal health stock are substitutes. Bed nets and malaria are an example. In areas with no malaria-parasite-carrying mosquitos, an individual's marginal gain from sleeping under a bed net brings almost no marginal benefit of preventing that individual from getting malaria. However, in areas with lots of malaria-parasite-carrying mosquitos, sleeping under a bed net reduces the marginal risk of contracting the disease.

In addition, health-seeking behavior x and exposure k are substitutes. For example, deworming pills could be very beneficial for a person that has low exposure to clean water and a clean environment (e.g., living close to dirty sewage). By contrast, they are not as beneficial for a person with access to clean water and a clean environment (high exposure k).

Assumption 6: Rich people have higher exposure and poor people have lower exposure to the communal health stock.

Results: The General Model

This section presents predictions from the model. Results are presented intuitively, with mathematical details appearing in the Appendix. This section examines how income inequality, residential segregation, and income segregation affects individual and communal health.

The Impact of Income Inequality Alone on an Individual's Health and the Communal Health Stock

In a community with no residential segregation, a decrease in income inequality, holding the average income in the community's constant, means that the poor's income, I_g, increases, while the rich's income I_r decreases. The model makes the following predictions:

- When income of the poor I_g increases (income inequality decreases), health-seeking behavior x^* of the poor increases because the poor have more money to afford health-seeking behavior. Thus, the poor are healthier.
- When the income of the rich I_r decreases (income inequality decreases), health-seeking behavior x^* of the rich decreases because the rich have less money to support their health-seeking behavior. Thus, the rich are less healthy.
- Communal health stock is the weighted average of individuals' health. When income inequality decreases (I_g rises and I_r falls) and residential segregation stays the same, the poor become healthier and the rich become less healthy. Therefore, what happens to the communal health stock when income inequality changes is ambiguous. In equilibrium, if the gain of the poor dominates the loss of the rich, the community health stock will increase when income inequality decreases. The reverse is true if the gain of the poor is not enough to compensate for the loss of the rich.

The Impact of Residential Segregation Alone on an Individual's Health and the Communal Health Stock

This subsection examines what happens to an individual's and a community's health when residential segregation changes. The model predicts that when a community goes from a perfectly integrated community to somewhat segregated, exposure (k) decreases in one group and increases for another group. When exposure k increases, its direct effect on an individual's health increases, increasing an individual's health because they now have better access to healthcare. In addition, higher exposure k makes people consume less health-seeking behavior x*, decreasing an individual's health. Collectively, it is unclear what happens to individuals' health as segregation increases. The communal health stock is the weighted average of individuals' health. Therefore, it is also unclear what happens to the communal health stock when residential segregation changes.

The Impact of Income Segregation on an Individual's Health and the Communal Health Stock

This section examines what happens to an individual's health and the communal health stock when income segregation changes. As defined earlier, income segregation is defined as a change in both income inequality and residential segregation. This scenario is a combination of the above two scenarios. In this case, income inequality is positively associated with residential segregation. When income inequality increases, residential segregation increases, and vice versa.

When income segregation decreases, income inequality and residential segregation decrease. A decrease in income inequality increases the poor's income, which in turns increases health-seeking behavior x^* of the poor because, intuitively, poor people can now afford more health goods. In addition, more health-seeking behavior x^* makes the poor healthier. So the health of the poor increases. A decrease in residential segregation increases exposure k of the poor to the communal health stock (better health facilities), which makes the poor healthier. However, higher exposure to communal health stock also makes the poor decrease their health-seeking behavior x* because they are less careful, decreasing their health. Collectively, it is unclear what happens to individuals' health as income segregation decreases.

Similarly, a decrease in income segregation means the exposure of the rich k_r and the income of the rich I_r decrease. Similar to the case of the poor, it is unclear what happens to the health of the rich. A decrease in income leads to lower health because the rich have less money to spend on health-seeking behavior. A decrease in exposure decreases the health of the rich because they have less access to good health-care facilities than before. However, a decrease in exposure increases the health of the rich because the rich are now less exposed to the communal health stock and, thus, more engaged in health-seeking behavior. As a result, the effect of a decrease in income segregation on the health of the rich is ambiguous.

The communal health stock is the weighted average of the health of rich and poor individuals. Therefore, what happens to the community's health when income segregation decreases is ambiguous.

Possible Equilibrium of the Health of the Poor and Health of the Rich

The model predicts that it is possible that the poor might get stuck in a low-health equilibrium and the rich stay in a high-health equilibrium. If so, the poor would need a lot of resources to get out of the low-health equilibrium to reach the high-health equilibrium. If the poor receive few resources to improve their health, and those resources are not sufficient to improve the health of the poor in a significant way, the poor would get stuck in the low-health equilibrium and will not reach the high-health equilibrium. This result is similar to the poverty trap idea, in which some communities or individuals get stuck in poverty if their income is not high enough.

Results: A Specific Model

A general model is helpful to guide us through different scenarios. However, the general model is ambiguous about what happens to an individual's health, and thus communal health stock, as income segregation changes. As a result, a more specific model that satisfies the general model's assumptions is often used in microeconomics to pin down the effect of income segregation on an individual's health. As before, this section presents the model's setup and states the predictions. The detailed mathematical derivations are shown in the Appendix.

Utility function: $U_i(c_i, h_i) = c_i^{\alpha} h_i^{\beta}$ where $\alpha, \beta \in (0,1)$. α and β are the preference parameters of c_i and h_i, respectively. The higher α is, the more this person prefers consumption goods ci. The higher β is, the more this person likes to be healthy. α and β take values in (0,1), which represents a diminishing marginal utility with respect to each activity holding the other constant. This utility function satisfies Assumption 1.

Health production function: $h_i = (x_i + k_i H)^{\gamma}$ with $0 < \gamma < 1$. The health production function satisfies Assumption 2 and Assumption 5.

The communal health stock: $H = \sum_i h_i / n$ with n as the number of people in the community. Thus, the community's health level is defined as an average of all individuals in the community.

Budget constraint: $c_i + px_i = I_i$. Our goal is to maximize utility with respect to health-seeking behavior xi and other consumption goods c_i. For now, we assume that individuals take the disease exposure k_i and the communal health stock H as exogenous factors. Based on his budget, health production function, and utility function, an individual maximizes his or her utility by allocating his or her income to health-seeking behavior or to other consumption goods.

This model predicts when exposure rate k and income increases, individual health increases. Thus, when income segregation decreases (income and exposure of the poor increase, and income and exposure of the rich decrease), the health of the poor increases, while that of the rich decreases. The model also predicts that the communal health stock increases when income segregation decreases.

Model Limitations

I recognize that this paper operates only within the rational choice model. A review by Dupas (2011) reviews some earlier works in the behavioral literature. In addition, this section points out other limitations of the model. When possible, I offer some intuitive predictions when the model changes to account for the limitations.

An Individual May Understand That His or Her Action Affects the Communal Health Stock

In this paper, I modeled how external factors (income segregation, the communal health stock, and exposure to the communal health stock) affect health and health-seeking behavior, which in turn affect an individual's health and the communal health stock. There may also be a scenario in which an individual internalizes the fact that his or her behavior will eventually affect the communal health stock, and the communal health stock will in turn affect his or her health. Note that in the model, an individual takes the communal health stock as a given and changes his or her behavior accordingly. In the second scenario, an individual understands that his or her behavior affects the communal health stock, which affects his or her health. In this case, their health-seeking behavior would be different from what the model in this paper predicts. To clearly know what the prediction would be, we would have to extend the current model.

Violation of Assumption 5

Additionally, under Assumption 5, this model only applies to cases where health-seeking behavior and the communal health stock are substitutes and where health-seeking behavior and the exposure are substitutes. In other words, a person's health-seeking behavior is more effective in increasing his or her health when the communal health stock or exposure to the communal health stock is low.

These assumptions don't always hold. It is certainly possible that they complement one another, instead of substitute. A person's health-seeking behavior may be less effective when the communal health stock or the exposure to the communal health stock is low. An example of when health-seeking behavior and the communal health stock are complementary is how taking vitamins to boost the immune system works better in an environment with clean water. However, in an environment where the water is polluted with chemicals and germs, the marginal effect of taking vitamins is small. In this scenario, a decrease in income segregation means that the poor have more money and higher exposure to the communal health stock. When the income of the poor increases, they have more money for such health-seeking behaviors as taking more medicine, resulting in an increase in their health. Exposure to the communal health stock increases an individual's health directly and indirectly via health-seeking behavior. Specifically, the direct effect implies that when the exposure to the communal health stock of the poor increases, the health of the poor increases because they now have better access to healthcare. The indirect effect of an increase in exposure suggests that a more exposed individual increases his or her health-seeking behavior because the health-seeking behavior is rewarded more than for a less exposed person. The increase in health-seeking behavior increases because exposure increases an individual's health. Thus, the health of the poor increases as the income segregation decreases.

When There Are More Than Two Types of Income

It is important to recognize that in reality, most societies have more than two groups of income, like the rich and the poor modeled in this paper. In fact, many countries have a large middle class. In addition, income distribution can be continuous. Ideally, I would model a situation that resembles the real world. In this paper, I modeled only two income groups so that I could focus on the novel interaction between an individual's health-seeking behavior and external factors in an environment with income segregation.

Intuitively speaking, let us say that a society has three income groups: poor, middle class, and rich. If the average income of a group is different from the average income of an egalitarian society, the model would suggest that this group faces a competing effect between exposure and health-seeking behavior as compared to the egalitarian society. Thus, whether the group's health increases or decreases depends on whether the exposure or health-seeking behavior effect is greater. Therefore, the effect of income segregation on the communal health stock is ambiguous, consistent with a world with only two income groups. The same intuition would apply if we had a continuous income distribution and segregation.

When Affluence Concentration Is Not Equivalent to Poverty Concentration

This paper analyzes the situation of income segregation in which the poor being highly segregated is equivalent to the rich being highly segregated. In other words, poverty concentration is equivalent to affluence concentration in this model. Thus, if income segregation exists, the exposure of the poor to the community's health decreases and that of the rich increases.

In practice, a community can have poverty-concentrated areas without affluence-concentrated areas. Additionally, in an environment with income segregation, either poverty concentration or affluence concentration may affect communicable disease transmission. Intuitively, the model suggests that when poverty concentration is not equivalent to affluence concentration, what happens to communicable disease transmission depends on where segregation happens (when the poor are clustered or when the rich are clustered). For example, income segregation happens because only the poor, not the rich, are segregated. The exposure of the poor to the communal health stock decreases, and that of the rich stays the same. The direct exposure effect makes the poor less heathy. However, low exposure means that the poor adopt more health-seeking behaviors, which increases their health. Overall, the model predicts ambiguous effects on the health of the poor. The health of the rich in the community with poverty concentration but no affluence concentration is the same as in the community where there is no such segregation. This is because the exposure of the rich does not change. Therefore, whether the communal health stock increases or decreases as a whole depends on the health of the poor. Similar results occur when there is an affluence concentration but no poverty concentration. The intuition would also be true if the income distribution is continuous.

Conclusion

This paper derives a novel theoretical model that examines the effect of income segregation on communicable disease transmission. The general model is ambiguous about how income segregation affects individual's and communal health. On the one hand, a decrease in income segregation increases the poor's income, which increases their health because the poor can now afford more health-seeking behavior. A decrease in income segregation also increases the poor's exposure to the communal health stock, which increases their health. Yet, higher exposure to the communal health stock is found to decrease the poor's health-seeking behavior, which reduces their health. Thus, a decrease in income segregation might increase or decrease the poor's health. Similarly, the general model is ambiguous about how income segregation affects the rich's health and thus, the communal health stock. The analysis replaces the general model with a more specific model predicts that overall, an increase in the poor's health due to higher income and exposure outweighs a decrease in the poor's health due to decreasing health-seeking behavior. Thus, a decrease in income segregation increases the health of the poor. Similarly, the health of the rich decreases as a result of decreasing income segregation. The specific model also predicts that the increase in health of the poor outweighs a decrease in the health of the rich, making the communal health increase. Furthermore, it is possible that the poor get stuck in a low-health equilibrium while the rich stay in a high-health equilibrium. For example, it could explain the HIV epidemic in Africa where it is mostly concentrated among the poor. Therefore, understanding how income segregation affects disease transmission will be helpful when policy makers want to know what policy to pursue and how to allocate appropriate resources to decrease the spread of communicable diseases.

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References:

Acevedo-Garcia, Dolores. 2000. "Residential Segregation and the Epidemiology of Infectious Diseases." *Social Science & Medicine* 51(8): 1143–61.

Bloch, Ethan. 2005. *The Real Numbers and Real Analysis*. Manuscript.

Cantwell, Michael F., Dixie E. Snider, George M. Cauthen, and Ida M. Onorato. 1994. "Epidemiology Of Tuberculosis In The United States, 1985 through 1992." *JAMA* 272(7): 535–39.

Collins, Chiquita, and David R. Williams. 1995. "Examining the Black-White Adult Mortality Disparity: The Role of Residential Segregation." *In Proceedings of the 1995 Public Health Conference on Records and Statistics.*

Cooper, Richard S. et al. 2001. "Relationship between Premature Mortality and Socioeconomic Factors in Black and White Populations of US Metropolitan Areas." *Public Health Reports* 116(5): 464–73.

Danziger, Sheldon, and Peter Gottschalk, eds. 1993. *Uneven Tides: Rising Inequality in America*. Russell Sage Foundation. http://www.jstor.org/stable/10.7758/9781610441469.

DeNavas, Carmen, and Robert W. Cleveland. 1996. *Money Income in the United States: 1995 (with Separate Data on Valuation of Noncash Benefits). Current Population Reports, Consumer Income*. https://eric.ed.gov/?id=ED403342 (July 9, 2017).

Dupas, Pascaline. 2011. "Health Behavior in Developing Countries." *Annual Review of Economics* 3(1): 425–49.

Fang, Jing, Shantha Madhavan, William Bosworth, and Michael H. Alderman. 1998.

"Residential Segregation and Mortality in New York City." *Social Science & Medicine* 47(4): 469–76.

Gaskin, Darrell J., Gniesha Y. Dinwiddie, Kitty S. Chan, and Rachael McCleary. 2012. "Residential Segregation and Disparities in Healthcare Services Utilization." *Medical Care Research and Review* 69(2): 158–75.

Greenberg, Michael, and Dona Schneider. 1994. "Violence in American Cities: Young Black Males in the Answer, but What Was the Question?" *Social Science & Medicine* 39(2): 179–87.

Guest, Avery M., Gunnar Almgren, and Jon M. Hussey. 1998. "The Ecology of Race and Socioeconomic Distress: Infant and Working-Age Mortality in Chicago." *Demography* 35(1): 23–34.

Jalan, Jyotsna, and E. Somanathan. 2008. "The Importance of Being Informed: Experimental Evidence on Demand for Environmental Quality." *Journal of Development Economics* 87(1): 14–28.

Jargowsky, Paul A. 1996. "Take the Money and Run: Economic Segregation in U.S. Metropolitan Areas." *American Sociological Review* 61(6): 984–98.

Kaplan, George A. et al. 1996. "Inequality in Income and Mortality in the United States: Analysis of Mortality and Potential Pathways." *BMJ* 312(7037): 999–1003.

Kawachi, Ichiro, Bruce P. Kennedy, Kimberly Lochner, and Deborah Prothrow-Stith. 1997. "Social Capital, Income Inequality, and Mortality." *American Journal of Public Health* 87(9): 1491–98.

LaVeist, Thomas A. 1992. "The Political Empowerment and Health Status of African-Americans: Mapping a New Territory." *American Journal of Sociology* 97(4): 1080–95.

———. 1993. "Segregation, Poverty, and Empowerment: Health Consequences for African Americans." *The Milbank Quarterly* 71(1): 41–64.

Lynch, John et al. 2004. "Is Income Inequality a Determinant of Population Health? Part 1. A Systematic Review." *The Milbank Quarterly* 82(1): 5–99.

Lynch, John W., George Davey Smith, George A. Kaplan, and James S. House. 2000. "Income Inequality and Mortality: Importance to Health of Individual Income, Psychosocial Environment, or Material Conditions." *BMJ* 320(7243): 1200–1204.

Madajewicz, Malgosia et al. 2007. "Can Information Alone Change Behavior? Response to Arsenic Contamination of Groundwater in Bangladesh." *Journal of Development Economics* 84(2): 731–54.

Massey, Douglas S. 1996. "The Age of Extremes: Concentrated Affluence and Poverty in the Twenty-First Century." *Demography* 33(4): 395–412.

Mellor, Jennifer M., and Jeffrey Milyo. 2002. "Income Inequality and Health Status in the United States: Evidence from the Current Population Survey." *The Journal of Human Resources* 37(3): 510–39.

Momota, Akira, Ken Tabata, and Koichi Futagami. 2005. "Infectious Disease and Preventive Behavior in an Overlapping Generations Model." *Journal of Economic Dynamics and Control* 29(10): 1673–1700.

Nuru-Jeter, Amani M., and Thomas A. LaVeist. 2011. "Racial Segregation, Income Inequality, and Mortality in US Metropolitan Areas." *Journal of Urban Health : Bulletin of the New York Academy of Medicine* 88(2): 270–82.

Polednak, Anthony P. 1991. "Black-White Differences in Infant Mortality in 38 Standard Metropolitan Statistical Areas." *American Journal of Public Health* 81(11): 1480–82.

———. 1993. "Poverty, Residential Segregation, and Black/White Mortality Ratios in Urban Areas." *Journal of Health Care for the Poor and Underserved* 4(4): 363–73.

———. 1996. "Segregation, Discrimination and Mortality in U.S. Blacks." *Ethnicity & Disease* 6(1–2): 99–108.

Pow, Choon-Piew. 2007. "Securing the 'Civilised' Enclaves: Gated Communities and the Moral Geographies of Exclusion in (Post-)Socialist Shanghai." *Urban Studies* 44(8): 1539–58.

Rabkin, Judith G., and Elmer L. Struening. 1976. "Live Events, Stress, and Illness." *Science* 194(4269): 1013–20.

Reardon, Sean F., and Kendra Bischoff. 2011. "Income Inequality and Income Segregation." *American Journal of Sociology* 116(4): 1092–1153.

Regidor, Enrique, M. Elisa Calle, Pedro Navarro, and Vicente Domínguez. 2003. "Trends in the Association between Average Income, Poverty and Income Inequality and Life Expectancy in Spain." *Social Science & Medicine* 56(5): 961–71.

Ross, Nancy A., Karla Nobrega, and James Dunn. 2001. "Income Segregation, Income Inequality and Mortality in North American Metropolitan Areas." *GeoJournal* 53(2): 117–24.

Sengupta, Somini. 2008. "Inside Gate, India's Good Life; Outside, the Servants' Slums." *The New York Times*. https://www.nytimes.com/2008/06/09/world/asia/09gated.html (July 9, 2017).

Silva, Maria Floresia Pessoa de Souza e. 2007. "Gated Communities: The New Ideal Way of Life in Natal, Brazil." *Housing Policy Debate* 18(3): 557–76.

Smith, George Davey. 1996. "Income Inequality and Mortality: Why Are They Related?" *BMJ* 312(7037): 987–88.

Spence, David P., Jason T. Hotchkiss, Christianna S. Williams, and Peter Davies. 1993. “Tuberculosis and Poverty.” *BMJ (Clinical research ed.)* 307(6907): 759–61.

Tarozzi, Alessandro et al. 2014. “Micro-Loans, Insecticide-Treated Bednets, and Malaria: Evidence from a Randomized Controlled Trial in Orissa, India.” *American Economic Review* 104(7): 1909–41.

Veenstra, Gerry. 2002. “Social Capital and Health (plus Wealth, Income Inequality and Regional Health Governance).” *Social Science & Medicine (1982)* 54(6): 849–68.

Wagstaff, Adam, and Eddy van Doorslaer. 2000. “Income Inequality and Health: What Does the Literature Tell Us?” *Annual Review of Public Health* 21(1): 543–67.

Waitzman, Norman J., and Ken R. Smith. 1998. “Separate but Lethal: The Effects of Economic Segregation on Mortality in Metropolitan America.” *The Milbank Quarterly* 76(3): 341–73.

Watson Tara. 2009. “Inequality and the Measurement of Residential Segregation by Income in American Neighborhoods.” *Review of Income and Wealth* 55(3): 820–44.

Wheeler, Christopher H., and Elizabeth A. La Jeunesse. 2008. “Trends in Neighborhood Income Inequality in the U.S.: 1980–2000.” *Journal of Regional Science* 48(5): 879–91.

Wildman, John. 2003. “Modelling Health, Income and Income Inequality: The Impact of Income Inequality on Health and Health Inequality.” *Journal of Health Economics* 22(4): 521–38.

Wilkinson, Richard G. 1996. *Unhealthy Societies : The Afflictions of Inequality / Richard Wilkinson*. London ; New York: Routledge.

Wu, Fulong. 2005. “Rediscovering the ‘Gate’ under Market Transition: From Work-Unit Compounds to Commodity Housing Enclaves.” *Housing Studies* 20(2): 235–54.

Yen, Irene H., and Leonard S. Syme. 1999. “The Social Environment and Health: A Discussion of the Epidemiologic Literature.” *Annual Review of Public Health* 20(1): 287–308.

Appendices:

Appendix on the Assumptions

Even though the utility function in the model’s setup section is denoted as $\mathrm{U}(c_i, h_i)$ for an individual *i*, the subscript *i* is abbreviated in the Appendix. Additionally, $\frac{\partial f}{\partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$ are denoted as f_x and f_{xy}, respectively.

Assumption 1: *Let* $U(c,h): (R^+U\{0\})^2 \rightarrow (R^+U\{0\})$ *be a twice differentiable function that satisfies the following conditions:* $U_c > 0, U_{cc} < 0, U_h > 0, U_{hh} < 0, U_{ch} > 0$. *In other words, utility increases at a decreasing rate with respect to a person's consumption c and health level h.*

Assumption 2: *Let* $h = f(x,k,H): R^{+3} \rightarrow R$ *be a twice differentiable function with respect to each variable and the second derivative with respect to each variable, which is continuous. Conditions:* $f_x > 0, f_k > 0, f_H > 0, f_{xx} < 0, f_{HH} < 0, f_{kk} < 0$

Assumption 3: *When x is sufficiently close to 0, denoted* $x \rightarrow 0$, *then* $\frac{U_h h_x}{U_c} > p$. *When x is sufficiently close to I/p, denoted* $x \rightarrow \frac{I}{p}$, *then* $\frac{U_h h_x}{U_c} < p$ → *technical assumption.*

Assumption 4: $x^*(k,I,p,H)$ *is a twice differentiable function with respect to each variable. In addition, the mixed partial derivatives of* x^* *are continuous* → *technical assumption*

Assumption 5: $h_{xH} < 0$ or $f_{xH} < 0$. Additionally, $h_{xk} < 0$ or $f_{xk} < 0$.

Assumption 6: $\partial k/\partial I > 0$

Appendix: Effects of Income Inequality

This section reports what happens to an individual's health and the communal health stock when income inequality decreases but residential segregation stays constant. Thus, income of the poor and the rich, I_g and I_r, change, and k_g and k_r stay constant. Mathematically, $\frac{\partial h^*}{\partial I_g}$, $\frac{\partial H^*}{\partial I_g}$, $\frac{\partial h^*}{\partial I_r}$, and $\frac{\partial H^*}{\partial I_r}$ are examined. By the proof of Theorem 0 below, an individual's health equilibrium exists. Thus, an individual's income can be written as:

$$h_i^* = f(x^*(I_i,p,k_i,H),k_i,H)$$

The partial derivative of h^* with respect to I_g and I_r are as followed:

$\frac{\partial h^*}{\partial I_g} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial I_g}$ and $\frac{\partial h^*}{\partial I_r} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial I_r}$

Now let us examine each term in the above expression.

- When income of the poor I_g increases (income inequality decreases), health-seeking behavior x^* increases because $\frac{\partial x^*}{\partial I_g} > 0$ (proof of Theorem 1). In addition, $f_x > 0$ by Assumption 2. Thus, $\frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial I_g} > 0$. Therefore, the health of the poor increases as income inequality decreases.
- When the income of the rich I_r decreases (income inequality decreases), health-seeking behavior x^* decreases because $\frac{\partial x^*}{\partial I_g} > 0$ (proof of Theorem 1 below). In addition, $f_x > 0$ by Assumption 2. Thus, $\frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial I_r} > 0$. Therefore, the health of the rich decreases as income inequality decreases.
- The communal health stock is the weighted average of individuals' health. When income inequality decreases and residential segregation stays the same, the poor become healthier

and the rich become less healthy. Therefore, what happens to the communal health stock when income inequality changes is ambiguous. In equilibrium, if the gain of the poor dominates the loss of the rich, the community health stock will increase when income inequality decreases. The reverse is true if the gain of the poor is not enough to compensate for the loss of the rich. Mathematically, the signs of $\partial H^*/\partial I_g$ and $\partial H^*/\partial I_r$ are ambiguous.

Appendix: Effects of Residential Segregation

First, the case when residential segregation changes and everyone has the same income is examined. By the proof of Theorem 0, an individual's health equilibrium exists. Thus, I can write:

$$\mathrm{h_i^* = f(x^*(I_i, p, k_i, H), k_i, H)}$$

The partial derivative of h^* with respect to $\mathrm{k_g}$ is as follows

$$\frac{\partial \mathrm{h}^*}{\partial \mathrm{k}} = \frac{\partial \mathrm{f}}{\partial \mathrm{x}^*}\frac{\partial \mathrm{x}^*}{\partial \mathrm{k}} + \frac{\partial \mathrm{f}}{\partial \mathrm{k}}$$

Now let us examine each term in the above expression.

- When a community goes from a perfectly integrated community to somewhat segregated, exposure k decreases for one group and increases for another group. When exposure k increases, its direct effect on individual's health increases: $\frac{\partial \mathrm{f}}{\partial \mathrm{k}} > 0$.
- Higher exposure (k) makes people consume less health-seeking behavior (in other words $\frac{\partial \mathrm{x}^*}{\partial \mathrm{k}} < 0$, proof of Theorem 4 in Appendix part b). Combined with Assumption 2 that assumes more health-seeking behavior would make people healthier directly, we have $\frac{\partial \mathrm{f}}{\partial \mathrm{x}^*}\frac{\partial \mathrm{x}^*}{\partial \mathrm{k}} < 0$.
- Collectively from the two terms, it is unclear what happens to individuals' health as segregation increases.
- The communal health stock is the weighted average of individuals' health. Therefore, it is also unclear what happens to the communal health stock when income segregation changes. Mathematically, the sign of $\partial H^*/\partial k$ is ambiguous.

Second, the residential segregation changes in an environment when income inequality exists. In other words, $\mathrm{k_g}$ and $\mathrm{k_r}$ change when $\mathrm{I_g} \neq I_r$. This case differs from the income segregation defined in this paper in a sense that in this scenario, it is not necessary that the poor have low exposure to the communal health stock and the rich have high exposure to the communal health stock.

The change in the health of the poor as residential segregation changes is:

$$\frac{\partial h_g^*}{\partial k_g} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial k_g} + \frac{\partial f}{\partial k_g}$$

The change in the health of the rich as residential segregation changes is:

$$\frac{\partial h_r^*}{\partial k_r} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial k_r} + \frac{\partial f}{\partial k_r}$$

Now let us examine each term in the above expression for the poor.

- When residential segregation decreases, exposure k_g increases for poor people and k_r decreases for rich people. The direct effect of a decrease in residential segregation on the health of the poor is $\frac{\partial f}{\partial k_g} > 0$. Thus, the health of the poor increases.
- Higher exposure k_g makes poor people consume less health-seeking behavior. In other words, $\frac{\partial x^*}{\partial k_g} < 0$ (proof of Theorem 4 in Appendix part b). Combined with Assumption 2, which assumes more health-seeking behavior would make people healthier directly, we have $\frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial k_g} < 0$. Because the poor decrease their health-seeking behavior, they become less healthy.
- Collectively from the two terms, it is unclear what happens to the health of the poor as residential segregation decreases, holding income inequality fixed.

Now let us examine each term in the above expression for the rich:

- When residential segregation decreases, exposure k_r decreases for the rich. Because $\frac{\partial f}{\partial k_r} > 0$, the direct effect suggests that a decrease in residential segregation decreases the health of the rich. Thus, the health of the rich decreases.
- Lower exposure k_r makes the rich consume more health-seeking behavior. In other words, $\frac{\partial x^*}{\partial k_r} < 0$ (proof of Theorem 4 in Appendix part b). Combined with Assumption 2, which assumes more (less) health-seeking behavior would make people healthier (sicker) directly, we have $\frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial k_r} < 0$. Because the rich increase their health-seeking behavior, they become healthier.
- Collectively from the two terms, it is unclear what happens to the health of the rich as residential segregation decreases, holding income inequality fixed.

From the above exercises, the communal health stock is the weighted average of rich and poor individuals' health. Therefore, it is also unclear what happens to the community's health when residential segregation decreases, holding income inequality fixed. Mathematically, the signs of $\frac{\partial H^*}{\partial k_g}$ and $\frac{\partial H^*}{\partial k_r}$ are ambiguous.

Appendix: The Impact of Income Segregation on an Individual's Health and the Communal Health Stock

This section examines what happens to an individual's health and the communal health stock when income segregation changes. Income segregation exists when income inequality is positively associated with residential segregation, that is, the poor have low exposure to the communal health stock and the rich have high exposure to the communal health stock. When income inequality increases, residential segregation increases, and vice versa. Thus, $\frac{\partial h^*}{\partial I_g}$, $\frac{\partial H^*}{\partial I_g}$, $\frac{\partial h^*}{\partial I_r}$ and $\frac{\partial H^*}{\partial I_r}$ are examined when $\frac{\partial k}{\partial I_g} > 0$ and $\frac{\partial k}{\partial I_r} > 0$.

By the proof of Theorem 0 in Appendix part b, an individual's health equilibrium exists and can be written as $h_i^* = f(x^*(I_i, p, k_i, H), k_i, H)$. When income segregation changes, the health of the poor changes in the following way:

$$\frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial I_g} + \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial k}\frac{\partial k}{\partial I_g} + \frac{\partial f}{\partial k}\frac{\partial k}{\partial I_g}$$

Now let us examine each term in the above expression.

- When income segregation decreases, the income of the poor increases and health-seeking behavior x^* increases because, intuitively, poor people can now afford more health goods (proof of Theorem 1 below). In addition, $f_x > 0$ by Assumption 2. Thus, $\frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial I_g} > 0$. In other words, more health-seeking behavior, x^*, makes a person healthier. The health of the poor increases because of the first term.
- Income segregation decreases mean that the poor have higher exposure (k) to the communal health stock. This makes the poor consume fewer health goods or decrease their health-seeking behavior (in other words $\frac{\partial x^*}{\partial k} < 0$, proof of Theorem 4 in Appendix part b). Combined with Assumptions 2 and 6, $\frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial k}\frac{\partial k}{\partial I_g} < 0$. This result implies the health of the poor decreases because of the second term.
- The third term $\frac{\partial f}{\partial k}\frac{\partial k}{\partial I_g} > 0$ by Assumptions 2 and 6.
- Collectively from the three terms, it is unclear what happens to the health of the poor as income segregation decreases.

As in the case of the poor, it is unclear what happens to the health of the rich when income segregation decreases. A decrease in income leads to lower health because the rich have less money to spend on health-seeking behavior. A decrease in exposure also decreases the health of the rich because the communal health stock is less effective on the health of the rich. However, a decrease in exposure makes the rich more engaged in health-seeking behavior, increasing their health.

Appendix: Possible Equilibrium of the Health of the Poor and the Rich

To examine the equilibria of the health of the poor and the rich, I examine the concavity of the communal health stock production function with respect to the communal health stock. The communal health stock function is said to be concave if it increases at the decreasing rate when the communal health stock increases, that is, when the communal stock increases, its function increases more when the communal health stock is low than when it is high. The communal health stock function is said to be convex if it increases at the decreasing rate when the communal health stock increases, that is, when the communal stock increases, its function increases more when the communal health stock is high than when it is low. Because the communal health stock is the weighted average of individuals' health, I first examine the concavity of individuals' health with respect to the communal health stock.

$$\frac{\partial h^*}{\partial H} = \frac{\partial f}{\partial x^*}\frac{\partial x^*}{\partial H} + \frac{\partial f}{\partial H}$$

$$\frac{\partial^2 h^*}{\partial H^2} = \frac{\partial^2 f}{\partial x^{*2}}\left(\frac{\partial x^*}{\partial H}\right)^2 + \frac{\partial^2 f}{\partial x^* \partial H}\,\frac{\partial x^*}{\partial H} + \frac{\partial f}{\partial x^*}\frac{\partial^2 x^*}{\partial H^2} + \frac{\partial^2 f}{\partial H \partial x^*}\,\frac{\partial x^*}{\partial H} + \frac{\partial^2 f}{\partial H^2}$$

Examining each term in the above expression:

$\frac{\partial^2 f}{\partial x^{*2}}\left(\frac{\partial x^*}{\partial H}\right)^2 < 0$ because $\frac{\partial^2 f}{\partial x^{*2}} < 0$ by Assumption 2.

$\frac{\partial^2 f}{\partial x^* \partial H}\,\frac{\partial x^*}{\partial H} > 0$ because $\frac{\partial^2 f}{\partial x^* \partial H} < 0$ by Assumption 5 and $\frac{\partial x^*}{\partial H} < 0$ (proof of Theorem 3 in Appendix part b).

$\frac{\partial f}{\partial x^*}\frac{\partial^2 x^*}{\partial H^2}$ cannot be determined because it depends on the sign of $\frac{\partial^2 x^*}{\partial H^2}$

$\frac{\partial^2 f}{\partial H^2} < 0$ by Assumption 2.

The model is ambiguous about whether the individual's health will increase or decrease at the increasing or decreasing rate in equilibrium. According to Assumption 2, the individual's health increases at the decreasing rate when the communal heath stock increases. However, when the communal health stock increases, individuals will decrease their health-seeking behavior *x**, which decreases their health. Depending on how much the decrease of *x** depreciates an individual's health and how much the increase of the communal health stock *H* improves his health, an individual's health production function may increase or decrease at the increasing or decreasing rate. Since the communal health stock is the weighted averaged of individuals' health, it is also ambiguous whether the communal health stock production function *m(H)* also increases or decrease at the increasing or decreasing rate.

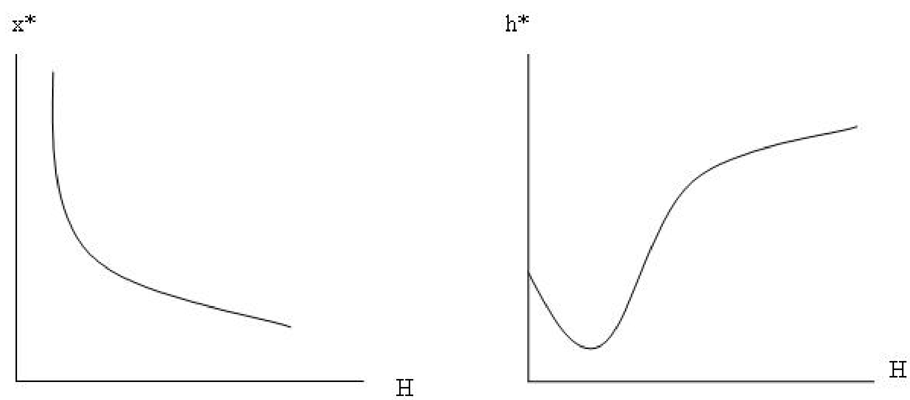

Figure 1: An example of the possible behavior of health production function h with respect to the community's health H.

An example of how an individual's health can decrease as the communal health stock increases can be seen in the TB outbreak in New York City in the early 1990s. Along with the decline of TB cases in the United States since the 1980s, the budget for TB prevention and treatment got smaller. However, immigrants from abroad once again introduced TB into New York City in the 1990s. Without adequate preparation, New York City experienced a TB outbreak. In addition, there are some diseases with a cyclical spread. For instance, syphilis has cycles of ten years in the United States and twenty years in Japan.[6] Since syphilis infections occur as a result of unprotected sexual intercourse, the incidence of syphilis is strongly associated with an individual's sexual behavior. As the spread of syphilis becomes more serious, people recognize the risk of infection, and they choose safe sexual behavior. This behavior then reduces the number of syphilis infections. In a low-risk environment, people are more engaged in risky sexual behaviors, and this results in higher syphilis prevalence.

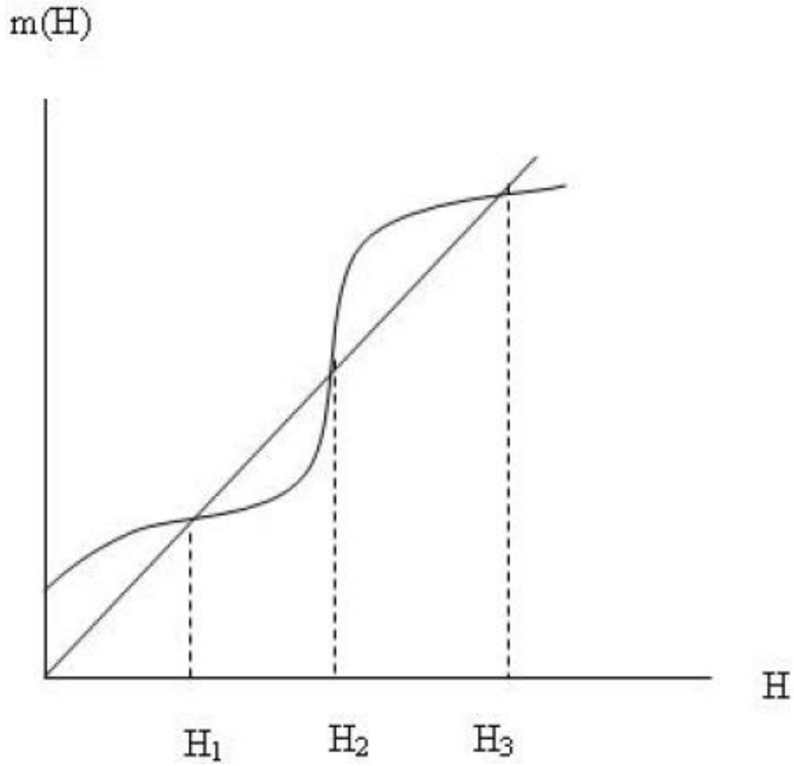

Figure 2: A possibility of multiple equilibria H.

The communal health stock production function m(H) can also exhibit many shapes. It is possible that the communal health stock reaches two stable multiple equilibria because it is an average of every individual's health in the community. If there are several stable equilibria, it is possible for a community to stay in one equilibrium while another community is in another equilibrium. For instance, when $H \rightarrow H_1$ in Figure 2, then the communal health stock will converge to H_1. If the communal health stock is high enough, it will reach a higher equilibrium health level $H = H_3$. If the communal health stock is not high enough, it will revert to the low equilibrium H_1. This brings up an issue of the reciprocal relationship between the communal health stock and an individual's health. A community that has many healthy individuals possesses a healthy environment. In exchange, this healthy environment will have a positive feedback to its people, which makes the entire community healthier. At the other end of the

[6] Momota, Tabata, and Futagami (2005).

spectrum, a community with many sick people will experience a less healthy environment. In exchange, this environment has a negative impact on the community's people, which results in a lower level of health for individuals. Unless there is a large change in the low health level in the community (from H_1 to H_3), every small change in the communal health stock will revert to its stable equilibrium H_1.

If a response health function of a disease in a country (m(H)) has multiple equilibria, it would be in the interest of policy makers and international organizations to know where the equilibria are so they can decide how much effort to put into a short-run disease prevention and treatment campaign. For instance, country A has high HIV/AIDS prevalence (low H1). Policy makers and international organizations want to implement a policy to improve the situation, but how many resources are enough to reach a new sustainable outcome? If policy makers stop somewhere in between H1 and H2, eventually the disease will revert back to its current situation, H1. On the other hand, if policy makers reduce the investment when the communal health stock is beyond H2 and switch back to a normal prevention and treatment scheme, then the disease's prevalence will reach its higher equilibrium, H3. As a result, knowing how many equilibria it has, where they are, and how long it takes to reach those equilibria are important questions for public health practitioners to consider. If the disease has already reached its highest equilibria, then a short-term campaign will not be sustainable in the long run unless policy makers decide to change the normal budget allocated to that disease permanently. The cyclic nature of syphilis in Japan and the United States demonstrates that money for syphilis intervention was apparently not enough for the disease to reach its new equilibrium. Every time there is a syphilis outbreak, the government increases its budget to fight the disease, but apparently it did not terminate the campaign on the right interval of H that allows syphilis to reach its new equilibrium. On the other hand, if "high" syphilis prevalence is already at a maximum level or the only equilibrium that syphilis can reach given a normal budget, then a temporary intervention is not going to result in a sustainable outcome unless the government can raise the budget permanently. Note that this permanent increase does not need to be the same amount as the temporary increase. The existence of multiple equilibria in this case comes solely from the change in concavity of the communal health stock production function. This change in concavity is a result of the decision to take more medication or not when the communal health stock increases. It can be because health goods are not as effective in a healthier environment as they are in a low health environment, so a small increase in the communal health stock leads to a large decrease in health goods. In general, a person's health is not only affected by that person's behavior but also by the environmental externality. Sometimes the external factor is so large that it outweighs an individual's behavior.

Appendix: A Specific Model

Differentiate U_i with respect to ci and xi using the Lagrange multiplier λ, we get the first order condition:

$$\frac{\partial U_i}{\partial c_i} = \alpha c_i^{\alpha-1} h_i^{\beta} = \alpha c_i^{\alpha-1} (x_i + k_i H)^{\beta\gamma} = \lambda \text{ (1)}$$

$$\frac{\partial U_i}{\partial x_i} = \beta\gamma c_i^{\alpha}(x_i + k_i H)^{\beta\gamma-1} = \lambda p \text{ (2)}$$

Dividing (1) with (2), we have: $\frac{\beta\gamma}{\alpha} \frac{c_i}{h_i+k_iH} = p$

Then we can write c_i in terms of x_i:

$$c_i = \frac{\alpha}{\beta\gamma} p(x_i + k_iH)$$

Substitution c_i into the budget constraint equation $c_i + px_i = I_i$, we get:

$$\frac{\alpha}{\beta\gamma} p(x_i) + px_i = I_i$$

$$px_i\left(\frac{\alpha}{\beta\gamma} + 1\right) = I_i - \frac{\alpha}{\beta\gamma} pk_iH$$

Solve x_i^* that maximizes the level of utility:

$$x_i^* = \frac{I_i}{p\left(\frac{\alpha}{\beta\gamma} + 1\right)} - \frac{\frac{\alpha}{\beta\gamma} k_iH}{\frac{\alpha}{\beta\gamma} + 1} = \frac{1}{\alpha + \beta\gamma}\left[\frac{\beta\gamma I_i}{p} - \alpha k_iH\right]$$

Thus,

$$h_i^* = \left[\frac{1}{\alpha + \beta\gamma}\left[\frac{\beta\gamma I_i}{p} - \alpha k_iH\right] + k_iH\right]^{\gamma} = \left[\frac{\beta\gamma \mathrm{I_i}}{\alpha p + \beta\gamma p} + \frac{\beta\gamma \mathrm{k_i H}}{\alpha + \beta\gamma}\right]^{\gamma}$$

a) Examine what happens to individual's health h_i^* when exposure rate k_i changes:

$$\frac{\partial h_i^*}{\partial k_i} = \gamma\left[\frac{\beta\gamma \mathrm{I_i}}{\alpha p + \beta\gamma p} + \frac{\beta\gamma \mathrm{k_i H}}{\alpha + \beta\gamma}\right]^{\gamma - 1} \frac{\beta\gamma \mathrm{H}}{\alpha + \beta\gamma} > 0$$

When exposure k increases, individual health increases.

b) Examine what happens to an individual's health h_i^* when income segregation decreases. The health of the poor changes based on the following:

$$\gamma\left[\frac{\beta\gamma \mathrm{I_i}}{\alpha p + \beta\gamma p} + \frac{\beta\gamma \mathrm{k_i H}}{\alpha + \beta\gamma}\right]^{\gamma - 1} \frac{\beta\gamma}{\alpha p + \beta\gamma p} + \gamma\left[\frac{\beta\gamma \mathrm{I_i}}{\alpha p + \beta\gamma p} + \frac{\beta\gamma \mathrm{k_i H}}{\alpha + \beta\gamma}\right]^{\gamma - 1} \frac{\beta\gamma \mathrm{H}}{\alpha + \beta\gamma} \frac{\partial k}{\partial I_g} > 0$$

When income increases, individual health increases. Since the income of the poor is smaller than the income of the rich and $\gamma < 1$, the increase in health of the poor is more than the decrease in health of the rich when income segregation decreases. Thus, the communal health stock increases in a community where there are as many poor people as rich people.

Appendix: Mathematical Background

<u>Theorem 2.2.1 (Intermediate Value Theorem):</u> *Let [a,b] ⊂ R be a closed bounded interval; and let $f: [a, b] \rightarrow R$ be a function. Suppose that f is continuous. If r ∈ R is strictly between $f(a)$ and $f(b)$, then there is some $c \in (a, b)$ such that $f(c) = r$.*

This theorem is the same as Theorem 3.5.2 in Real Analysis by Bloch (2005).

Theorem 2.2.2 (Mean Value Theorem): *Let* $[a, b] \subset R$ *be a non-degenerate closed bounded interval, and let* $f: [a, b] \to R$ *be a function. Suppose that* f *is continuous on* $[a, b]$ *and differentiable on* (a, b). *There is some* $c \in (a, b)$ *such that* $f'(c) = \frac{f(b)-f(a)}{b-a}$

This theorem is the same as Theorem 4.4.4 in Real Analysis by Bloch (2005).

Theorem 2.2.3: *Let* $I \to R$ *be an open interval, let* $c \in I$, *and let* $f: I \to R$ *be a function. Then* f *is a continuous function at c if and only if* $\lim_{x \to c} f(x)$ *exists and* $\lim_{x \to c} f(x) = f(c)$.
Same as theorem 3.3.2 by Bloch (2005).

Theorem 2.2.4(Clairaut's Theorem): *Suppose f is defined on disk D that contains the point (a,b). If the function* f_{xy}*and* f_{yx} *are both continuous on D, then* $f_{xy}(a, b) = f_{yx}(a, b)$

Theorem 2.2.5: *Let* $I \subset R$ *be an open interval, and let* $f: I \to R$ *be a function. Then the following are equivalent:*

a) *If* $a, b \in I$ *and a<b, then* $f(ta + (1-t)b) \leq tf(a) + (1-t)f(\, b)$ *for all* $t \in [0,1]$ *(Function is Convex).*
b) *Suppose that f is differentiable. Then the function is convex if and only if* $f''(x) \geq 0$ *for all* $x \in I$.

This theorem in equivalent to theorems 4.6.2 and 4.6.4 in Bloch (2005).

Theorem 2.2.6:*Let* $I \subset R$ *be an open interval, and let* $f: I \to R$ *be a function. Then the following are equivalent:*

a) *If* $a, b \in I$ *and* $a < b$, *then* $f(ta + (1-t)b) > tf(a) + (1-t)f(\, b)$ *for all* $t \in [0,1]$ *(Function is Concave).*
b) *Suppose that f is differentiable. Then the function is concave if and only if* $f''(x) < 0$ *for all* $x \in I$.

Appendix: Theorem of This paper.

Let V(x)= U(c, h(x))
We have $I = c + px$. Thus $x = \frac{I-c}{p}$

Theorem 0: *Let V(x)=U(c,h(x)) be a continuous function on* $[0, \frac{I}{p}]$ *and twice differentiate function on* $\left(0, \frac{I}{p}\right)$. *Suppose that V(x) satisfies Assumptions 1, 2, and 3. Then there exists a unique* $x^* \in (0, \frac{I}{p})$ *such that* $V(x^*)$ *is a global maximum on* $[0, \frac{I}{p}]$

Proof:

By the rule, we have:

$$\frac{\partial V}{\partial x} = \frac{\partial U}{\partial c}\frac{\partial c}{\partial x} + \frac{\partial U}{\partial h}\frac{\partial h}{\partial x} = -\frac{p\partial U}{\partial c} + \frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$$

Second derivative:
$\frac{\partial^2 V}{\partial x^2} = \frac{p^2 \partial^2 U}{\partial c^2} - p\frac{\partial^2 U}{\partial c \partial h}\frac{\partial h}{\partial x} - p\frac{\partial^2 U}{\partial h \partial c} + \frac{\partial^2 U}{\partial h^2}\left(\frac{\partial h}{\partial x}\right)^2 + \frac{\partial U}{\partial h}\frac{\partial^2 h}{\partial x^2} < 0$ (because each term is less than 0)

As $x \to 0, V_x \to 0$ by Assumption 3. As $x \to \frac{I}{p}, V_x < 0$ by Assumption 3. Therefore, there exists $x^* \in (0, \frac{I}{p})$such that $V_x(x^*) = 0$ by the Intermediate Value Theorem.

Since $V_{xx} < 0$ for all $x \in \left(0, \frac{I}{p}\right)$, $x^*$is unique.

For $x_1 < x^*$for all $x_1 \in (0, x^*), V_x(x_1) > V_x(x^*) = 0$. Thus, $V(x_1) < V(x^*)$.

For $x_2 > x^*$for all $x_2 \in \left(x^*, \frac{I}{p}\right), V_x(x_2) < V_x(x^*) = 0$. Thus, $V(x_1) < V(x^*)$.

Therefore, $V(x^*)$is the unique global maximum on $\left(0, \frac{I}{p}\right)$and $x^*$is also unique.

Therefore, there exists an $a > 0$ such that as $x \to 0, V(x) \leq V(x^*) - a$. In addition, there exists $b > 0$ such that when $x \to \frac{I}{p}$, then $V(x) \leq V(x^*) - b$.

Because $V(x)$is a continuous function on $\left[0, \frac{I}{p}\right], V(x)$is continuous at 0 and at $\frac{I}{p}$. Hence $V(0) = \lim_{x \to \frac{I}{p}} V(x)$ by Theorem 2.2.3. Therefore, $V(0) \leq V(x^*) - a < V(x^*)$, and $V\left(\frac{I}{p}\right) \leq V(x^*) - b < V(x^*)$. We conclude that $V(x^*)$is the unique global maximum of $V(x)$on $[0, \frac{I}{p}]$

Theorem 1: *As I increases, so does x*.*

Proof: Let $I_1$and I_2be two different incomes and suppose that $I_1 < I_2$. Then $V(x)$ has a maximum at x_1^* on $[0, \frac{I_1}{p}]$ and a maximum at x_3^* on $[0, \frac{I_2}{p}]$ by the proof of theorem 2.2.7.

Recall the first derivative:

$$\frac{\partial V}{\partial x} = -\frac{p \partial U}{\partial c} + \frac{\partial U}{\partial x}$$

Consider the interval $[0, \frac{I_1}{p}]$ and let x_2^*be the point where $V(x)$ achieves its maximum on $[0, \frac{I_1}{p}]$. For a given x, we always have $-\frac{p\partial U}{\partial c}|I = I_1 < -\frac{p\partial U}{\partial c}|I = I_2$ because $c_1 = I_1 - x < I_2 - x = c_2$. In addition, for a given x, $\frac{\partial U}{\partial h}\frac{\partial h}{\partial x}|I = I_1 = \frac{\partial U}{\partial h}\frac{\partial h}{\partial x}|I = I_2$. Therefore, on interval $\left[0, \frac{\partial I_1}{\partial p}\right], \frac{\partial V}{\partial x}|I = I_1 < \frac{\partial V}{\partial x}|I = I_2$. Follow the similar proof as that of Theorem 2.2.8, we have $x_2^* > x_1^*$.

Since $\left[0, \frac{I_1}{p}\right] \subset [0, \frac{I_2}{p}]$, so $x_3^* \geq x_2^*$. As a result, $x_3^* > x_1^*$. WE prove that as income increases, so is x^*.

Theorem 2: *Holding k fixed, as I increases, h* increases. In other words,* $\frac{\partial h^*}{\partial I} > 0$

Proof:

$\frac{\partial h^*}{\partial I} = \frac{\partial f}{\partial x^*} \times \frac{\partial x^*}{\partial I}$

By the theorem 1: $\frac{\partial x^*}{\partial I}$ >0, by the assumption 2: $\frac{\partial f}{\partial x^*} > 0$. Thus, an individual's health increases as income increases.

Theorem 3: *Holding I fixed, as H increases, x* decreases. In other words,* $\frac{\partial x^*}{\partial H} < 0$.

Proof: Recall the first derivative of V with respect to x:

$$\frac{\partial V}{\partial x} = -\frac{p\partial U}{\partial c} + \frac{\partial U}{\partial h}\frac{\partial h}{\partial x}$$

Then $\frac{\partial^2 V}{\partial x \partial H} = -p\frac{\partial^2 U}{\partial c \partial h}\frac{\partial h}{\partial H} + \frac{\partial^2 U}{\partial h^2}\frac{\partial h}{\partial H}\frac{\partial h}{\partial x} + \frac{\partial U}{\partial h}\frac{\partial^2 h}{\partial x \partial H}$ (2.2.7)

Equation 2.2.7 is always less than 0 because $-p\frac{\partial^2 U}{\partial c \partial h} < 0$ and $\frac{\partial^2 U}{\partial^2 h} < 0$ by Assumption 1. And $\frac{\partial^2 h}{\partial x \partial H} < 0$ by Assumption 5. Therefore, $\frac{\partial V}{\partial x}$ decreases as H increases.

Let x_1^* and x_2^* denote the value at which the utility reaches its maximum when $H = H_1$ and $H = H_2$, respectively, with $H_1 < H_2$. Therefore, for every x, $\frac{dV}{dx}|H = H_1 > \frac{dV}{dx}|H = H_2$for $H_1 < H_2$. Therefore at x_1^*, $\frac{dV}{dx}\Big|H = H_2 < \frac{dV}{dx}\Big|H = H_1 = 0$. Thus, $x_2^* < x_1^*$.

Theorem 4: *Holding I fixed, as k increases, x* decreases. In other words,* $\frac{\partial x^*}{\partial k} < 0$

Proof:

$$\frac{\partial V}{\partial x} = \frac{\partial U}{\partial c} \times \frac{\partial c}{\partial x} + \frac{\partial U}{\partial h} \times \frac{\partial h}{\partial x} = -\frac{p\partial U}{\partial c} + \frac{\partial U}{\partial h} \times \frac{\partial h}{\partial x}$$

$\frac{\partial^2 V}{\partial x \partial k} = \frac{\partial^2 U}{\partial h^2} \times \frac{\partial h}{\partial k} \times \frac{\partial h}{\partial x}$<0

Because of the assumption $\frac{\partial^2 U}{\partial h^2} < 0$, and $\frac{\partial h}{\partial k} > 0$, and $\frac{\partial h}{\partial x} > 0$. Following the same proof as Theorem 3, as k increases, x* decreases.

Theorem 5: *Holding I fixed, as k increases, the effect on an individual's health h is ambiguous. In other words, the sign of* $\frac{\partial h^*}{\partial k}$ *is ambiguous.*

Proof:

$$\frac{\partial h^*}{\partial k} = \frac{\partial f}{\partial x^*} \times \frac{\partial x^*}{\partial k} + \frac{\partial f}{\partial k}$$

Since $\frac{\partial f}{\partial k} > 0$, $\frac{\partial f}{\partial x^*} > 0$, and $\frac{\partial x^*}{\partial k} < 0$, so $\frac{\partial h^*}{\partial k}$ is unclear.